

# Integrated Resource Analysis with Processes

Eric Kemp-Benedict

November 25, 2004 (last modified May 7, 2006)

## Introduction

At the heart of many sustainability studies is the need to track the way natural and other resources are being used as they flow through or are transformed by human and natural systems. There are generally many ways in which resources can be used, and the task is to find a pattern of use that stays within sustainability constraints while also satisfying other goals. This paper presents a way to carry out such a study.

The approach described in this paper assumes that the ways in which materials are transformed in societies can be approximated as weighted sums of linear processes (so, e.g., doubling the output from a process requires a doubling of all of the inputs). In this case, it is possible to express this key sustainability question within a linear programming framework. Also, many resource-use analyses feature precisely this assumption – for example, that different stoves have fixed technical coefficients, and the net effect of all of the stoves in society, in terms of material inputs and outputs can be determined by a weighted sum across all the technical coefficients of all stoves.

## Balancing Inputs and Outputs: The Core Equations

In the approach described in this paper, societal and environmental needs are met through linear processes with fixed proportions of inputs and outputs, as illustrated in Illustration 1. The processes are combined within a linear program.



Illustration 1. A basic process

The core equations are very similar to those of an input-output (I-O) matrix. As in the I-O approach, outputs might go to final consumption or intermediate consumption (that is, they may be an input to a downstream process). In addition, in the scheme presented here, outputs may act as pollutants or be used for ecosystem services, in which case they are not used for either final or intermediate consumption.

Inputs may be *primary* (that is, raw feedstocks) or *secondary* (that is, an input may be an output from an upstream process). One output from each process is identified as the main output. This output has a coefficient of 1, to which all other inputs and outputs are normalized. Given the *scale* of output from a process, the level of all inputs and outputs can be determined by multiplying the scale by the coefficient.

The first core equation maintains a balance over all process outputs. It can be written,

$$\sum_p \left( O_{op} - \sum_i \theta_{io} \cdot I_{ip} \right) \cdot S_p - P_o^{\text{net}} = D_o \quad , \quad (1)$$

where

$S_p$  is the scale of output from process  $p$

$P_o^{\text{net}}$  is net production not going to either requirements or intermediate use

$D_o$  is final demand for output  $o$

$\theta_{io}$  is equal to 1 if output  $o$  can be used as input  $i$ , 0 otherwise

$O_{op}$  is the coefficient for output  $o$  from process  $p$  (equal to 1 for primary output)

$I_{ip}$  is the coefficient for input  $i$  from process  $p$

Input and output coefficients in Equation (1) are normalized so that the scale variable  $S_p$  has the units of the main output. So, for example, if the process producing electricity from natural gas,  $S_p$  might have units of kWh, while the input coefficient for natural gas might have units of cubic meters per kWh.

The second core equation accounts for primary inputs (that is, process inputs that are not the outputs from another process). This can be expressed as,

$$\sum_p I_{ip} \cdot S_p - C_i^{\text{net}} = 0 \quad \text{for all primary inputs } i \quad , \quad (2)$$

where  $C_i^{\text{net}}$  is net consumption of primary input  $i$ .

The final core equation constrains certain outputs to not be supplied in excess. An example of a situation where one output can be produced in excess, while another cannot, is combined heat and power (CHP), in which electricity production should not exceed total requirements, but heat production may exceed demand. This equation is written simply,

$$P_o^{\text{net}} = 0 \quad \text{for outputs } o \text{ that are never produced in excess} \quad . \quad (3)$$

## Sustainability Constraints

The requirement of sustainability often manifests itself in the form of constraints on the magnitude of resource or pollutant flows. For renewable resources, extraction of the resource should not exceed the rate of replenishment, and may have to be well below that level in order to maintain ecosystem health, while the flow of pollutants must not exceed the buffering capacity of natural systems. This can be expressed as,

$$\sum_i \lambda_{ik} \cdot C_i^{\text{net}} + \sum_o \mu_{ok} \cdot P_o^{\text{net}} \leq K_k \quad , \quad (4)$$

where

$\lambda_{ik}$  is the resource coefficient for primary input  $i$  for constraint  $k$

$\mu_{ok}$  is the production coefficient for output  $o$  for constraint  $k$

$K_k$  is the constraint value for constraint  $k$

For example, if  $\text{CO}_2$  is an unwanted output from some process (and given the index  $o = 1$ ), then it will have a nonzero value for  $P_1^{\text{net}}$ . A carbon emission constraint  $K_1$  could be specified as a maximum level of carbon emissions. Then the coefficient  $\mu_{11}$  would be set equal to 1. Since no primary input is involved in the constraint,  $\lambda_{i1} = 0$  for all inputs  $i$ .

In Equation (4), only unwanted outputs are considered. There could be a requirement that some desirable output *exceed* a certain level (for example, a minimal flow requirement for an aquatic ecosystem). Also, some constraints may be equalities – for example, total land area might be set to a fixed value.

## Decision Rules

Unlike in an I-O approach, in the approach presented in this document there are multiple ways of meeting final demands. To determine which combination of processes is used to meet demands, a decision rule must be added to the equations, corresponding to the objective function of the linear program (or to several objective functions, in multi-criteria variants of linear programming). There are many ways to specify a decision rule. One possible approach will be presented here.

### A Linear Goal Programming Approach

In sustainable development studies, it is frequently useful to ask how much effort must be made to depart from some reference pattern of consumption and production in order to meet sustainability constraints. The reference pattern might be a “business-as-usual” pattern of consumption, or some other reference, such as a socially desired (but potentially environmentally unsustainable) pattern. This kind of question can be addressed within a linear goal programming framework. In linear goal programming, a solution is found by minimizing the weighted sum of the deviations away from some target value of some of the solution variables.

To implement this approach within the system of equations presented above (Equations 1-4), it is convenient to group processes with the same main output into *categories*, labeled by  $c$ . The reference combination of processes is set by setting the share of each processes within each category. Shares sum to one over processes within a category. This can be expressed via,

$$\begin{aligned} X_{pc} \cdot S_p - f_{pc} \cdot v_c + X_{pc} \cdot \delta_c^+ &\geq 0 \\ X_{pc} \cdot S_p - f_{pc} \cdot v_c - X_{pc} \cdot \delta_c^- &\leq 0 \end{aligned} \quad (5)$$

where the objective function is to minimize a weighted sum of the deviations  $\delta_c^\pm$ ,

$$\text{Minimize } \sum_c w_c (\delta_c^+ + \delta_c^-) \quad (6)$$

where the  $w_c$  are the weights. In Equation (5),

$v_c$  is total production of main output from category  $c$  (ensures consistency of LP)

$\delta_c^+$ ,  $\delta_c^-$  are deviations from the reference pattern

$X_{pc}$  is equal to 1 if process  $p$  is in category  $c$ , 0 otherwise

$f_{pc}$  is the share of process  $p$  in category  $c$

## Sustainability Exploration

Equations 1-6 provide a complete framework for exploring a certain class of sustainability problems: in which resource flows are constrained, transformation processes can be

represented as linear, and the question of interest is how much effort is required to depart from a reference combination of processes. Within this framework, sustainability issues can be explored in at least five ways, by

1. Changing final demand,
2. Changing the weights on different categories of outputs,
3. Modifying constraints on inputs and outputs,
4. Redefining a waste product as a useful input,
5. Changing the nature of "demand."

Most of these possibilities are straightforward within this framework. For example, changing the weights on different categories is a typical way to use a linear goal program for multi-criteria analysis. In cases where there is no obvious basis of comparison between different resources or pollutants (for example, a constraint on CO<sub>2</sub> emissions vs. a constraint on water use), the weights can be changed to match possible preferences of different actors.

Possibilities 4 and 5 require some explanation. In the case of 4, new processes can be introduced that use what once were waste outputs for a useful end (either for people or the environment).

In the case of 5, it is possible to shift what is conventionally considered "final demand" into processes, while creating higher-level types of output and demand. For example, while conventionally energy is included within final demand (or, better, *useful* energy), if substantial changes in lifestyles are anticipated, then a more appropriate output might be "recreation," and different processes might use different amounts of energy and other inputs to meet that requirement.